

# Multiple-Attribute Decision Making with Estimation of Preferences of Multiple Evaluators

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**Abstract**—A method is proposed to solve a class of multiple-attribute decision making problems where the alternatives are evaluated by two or more evaluators different from the decision maker. In multiple-attribute decision making, the decision maker must find the best one among a set of Pareto optimal solutions. Usually, this is done based on the preference of the decision maker. However, in the problems treated here, the alternatives are evaluated not by the decision maker but by other people. For example, in public services, the service provider is the decision maker who is to choose and provide the high-quality services, while the quality of services is evaluated by the clients. Hardly any of the existing methods addresses this type of problems. This paper proposes an interactive method for estimation of multiple evaluators' preferences, their reasonable unification and finding the best solution. An example illustrates the validity and efficiency of the proposed method.

**Keywords**—*multi-objective optimization; interactive methods; pairwise comparison; linguistic values; public services.*

## I. INTRODUCTION

In decision making problems, the decision maker must find the best one among a number of alternatives. Each alternative is characterized and evaluated by its attributes. Usually each alternative has more than one attribute, which makes the problem a *multiple*-attribute decision making problem. To this kind of problems, a set of Pareto optimal solutions exist each of which is optimal in the sense that there are no other alternatives superior to it in every attribute. The decision maker must choose a single solution among the Pareto optimal solutions based on his/her preference.

There have been proposed a number of solution methods for multiple-attribute decision making problems or multi-objective optimization problems [1]. Some methods incorporate the decision maker's preference before optimization process [2], [3], where naturally the decision maker must well understand his/her own preference so that it can be suitably expressed in the objective function to be optimized. Recently a large number of methods have been proposed which aim at finding as many Pareto optimal solutions as possible using evolutionary computations [4]. The decision maker picks up the best solution from the found Pareto optimal solutions taking his/her preference into consideration. Here the preference does not need to be explicitly expressed. In another category of

solution methods called 'interactive methods' the preference is estimated through rounds of interactions and is used to find the single best solution [5]. In each round of interaction, a question is presented to the decision maker and his/her answer is obtained which is then used to update his/her preference estimate.

In some problems, the alternatives are evaluated not by the decision maker but by other people. For example, in public services, the service provider is the decision maker who is responsible for choosing and providing high-quality services, while the quality of services is evaluated by the clients. Hardly any of the existing methods addresses this type of problems. This paper proposes a method to solve these problems. In these problems, the best solution is determined by the evaluators' preferences but not by the decision maker's preference. Usually the decision maker (service provider) does not well realize the evaluators' (clients') preferences. Therefore estimation of the preferences is essential. In this respect, the interactive methods are most promising and will be used here.

The problems treated here have two major issues to be solved. One is how to estimate the evaluators' (clients') preferences, and the other is how to find the single solution based on preferences of *multiple* evaluators. In order to solve the first issue, an interactive method is employed here. However, in our target problems, in order to estimate the preferences of the evaluators, the decision maker asks questions to the evaluators (clients), and unlike the decision maker, the evaluators might be uncooperative in answering questions or unable to give accurate and consistent answers. Interactions must be designed so that the evaluators can easily answer to the questions raised by the decision maker. The second issue demands that the preferences of individual evaluators can be compared to each other so that they can be reasonably unified into a single collective preference. The preference is, in its nature, a subjective measure of goodness. However, when a compromise is needed between the conflicting preferences of different people, it is necessary to 'trade off' their preferences, which requires the subjective preferences to be represented in an objectively comparable way. The proposed method solves the above two issues and provides the best solutions to given

problems. The authors have already proposed a method for estimating the preference function of a single evaluator [6] and have combined it in a rather straightforward way to deal with multiple evaluators [7]. The paper proposes a new method for preference estimation and, based on that, a reasonable and realistic way to unify the preferences of multiple people in order to find the best solution.

The paper is organized as follows. In the next section, the problem to be solved is stated together with its associated assumptions. Section III describes the proposed method, which includes representation of individual and collective preferences, their estimation through interactions, and finding the best solution. The method is evaluated by an example in Section IV, which is then followed by conclusions.

## II. PROBLEM STATEMENT

There is a set of a finite number  $n$  of alternatives,  $X = \{x_1, \dots, x_n\}$ . Each alternative  $x$  has  $p$  attributes, and the attributes are expressed numerically by  $p$  criterion functions  $f_1(x), \dots, f_p(x)$ . It is assumed without loss of generality that the problem is a minimization problem i.e. smaller criterion value  $f_j$  is better for every  $j$ . The decision maker knows  $X$  and criterion values  $f_j(x)$  for all  $x \in X$ . There are  $m$  evaluators. Evaluator  $i$  has his/her own (individual) preference function  $v_i(x)$  that indicates how good an alternative  $x$  is for him/her. The preference functions are unknown to the decision maker. The evaluators do not necessarily know their own preference function explicitly, but it is assumed that they can give correct and consistent answers to the questions raised by the decision maker during the interaction process.

We want to find the best solution  $x^*$  in  $X$ . Here by the best solution it is meant that the solution gives the maximum collective preference which is appropriately defined based on the individual preferences of  $m$  evaluators.

## III. MULTIPLE-ATTRIBUTE DECISION MAKING WITH ESTIMATION OF PREFERENCES OF MULTIPLE EVALUATORS

### A. Representation of preferences

Usually people cannot describe their own individual preference function explicitly. However, when they are shown any pair of alternatives, it is not difficult for them to tell which one in the pair is better [8], [9], [10], and this judgment is done based on the attributes of the alternatives. Therefore, it is safe to assume that the preference functions are real-valued functions of the attribute criteria values. Usually a preference function is defined so that it gives a larger value for a better alternative. Since we treat here a minimization problem of  $f_j, j = 1, \dots, p$ , the individual preference  $v_i$  of any evaluator  $i$  is a monotonically decreasing function of  $f_1, \dots, f_p$ . Also we assume that  $v_i$  is a smoothly changing function of  $f_j, j = 1, \dots, p$ . Because each  $f_j$  is a function of  $x$ ,  $v_i$  is also a function of  $x$ .

The individual preference itself is a subjective evaluation and depends on the individual who evaluates the alternatives. In order to combine them into a single collective preference that reasonably indicates how good any alternative is for

the evaluators as a whole, the preference values of different evaluators must be comparable. This requires for the real-valued preferences of all the evaluators to be represented on the same scale. On the other hand, one cannot tell his/her preference of an alternative as an exact real value. At best, he/she can express the preference as linguistic values such as 'very good', 'good', 'neutral', 'bad', and 'very bad'. Although these are vague and imprecise, they can be useful to compare preferences of different people. For example, we can know that the alternative  $x$  is more readily accepted by evaluator  $i$  than evaluator  $j$  when evaluator  $i$  thinks the alternative  $x$  'very good' whereas evaluator  $j$  rates it as 'good'. Therefore these linguistic values are used in the interactions between the decision maker and the evaluators for two reasons. One is that it makes it easier for the evaluators to express their preference values. The other is that they can express the preferences of different people on the same (linguistic) scale. These linguistic values are mapped to numerical values so that we can treat the preference functions as real-valued functions. For example, the linguistic values 'very good', 'good', 'neutral', 'bad', and 'very bad' can be represented by numerical values 5, 4, 3, 2 and 1, respectively.

We do not know the function form of  $v_i$  as a function of  $f_1, \dots, f_p$ . However, since  $v_i$  is assumed to be a monotonically and smoothly changing function of  $f_j$ , it is expected that it can be well approximated by its tangential hyper-plane in the  $f$ -space.

$$v_i = c_{i0} + c_{i1}f_1 + \dots + c_{ip}f_p, \quad (1)$$

where  $c_{i1}, \dots, c_{ip}$  are all negative because  $v_i$  is a monotonically decreasing function. Estimation of preference functions is now reduced to estimation of parameters  $c_{ij}, j = 0, \dots, p$ .

The collective preference  $V$  that somehow represents the preference of all the evaluators can be defined based on  $v_i$ . One possible way is to define  $V$  as a sum of the preferences of all the evaluators,

$$V(x) = \sum_{i=1}^m v_i(x). \quad (2)$$

Considering eq. (1),  $V$  defined in eq. (2) is a weighted sum of criterion functions  $f_j, j = 1, \dots, p$ , where the weighting coefficients are all negative, and thus maximizing this gives a Pareto optimal solution as the ordinary weighting method for multiobjective optimization problems [1]. Maximizing this collective preference function is equivalent to finding the solution that is best for the 'average' evaluator. Individual preference of this solution may vary from evaluator to evaluator, and some evaluators may find this solution unsatisfactory. Another option is based on the min-max idea and to define  $V$  as the minimum (worst) of the preferences of the  $m$  evaluators:

$$V(x) = \min_{i \in \{1, \dots, m\}} v_i(x). \quad (3)$$

This collective preference function focuses on evaluators having low preferences and therefore leads to a 'fair' solution.

## B. Estimation of preferences

The first major issue should be solved by devising a good method to estimate preferences without putting much burden on the evaluators.

The decision maker knows the values of  $f_1(x), \dots, f_p(x)$  for all  $x \in X$ . If the evaluator could provide his/her  $v_i(x)$  values for  $p+1$  or more distinct alternatives  $x$ , the parameters  $c_{ij}, j = 0, \dots, p$ , would be easily determined by the least square method. However, it is usually not easy to tell precisely and accurately the preferences of alternatives. On the other hand, it is easier to accurately distinguish two alternatives by the preferences (pairwise comparison), and also to express the preference as a linguistic value.

Based on the above, the decision maker presents, at each round of interaction, a pair of alternatives  $x_{q1}$  and  $x_{q2}$  both in  $X$  to every evaluator, inquires which is preferable and also asks the evaluator to rate the better alternative using a set of predetermined linguistic values, e.g. {'very good', 'good', 'neutral', 'bad', 'very bad'}. Then each evaluator gives his/her answer. The preference parameters  $c_{ij}, i = 1, \dots, m; j = 0, \dots, p$ , are updated every round of interaction. This update is performed in two steps.

Suppose that evaluator  $i$  has answered that  $x_{q1}$  is better, then the first step proceeds as follows. From the answer we know that  $v_i(x_{q1}) > v_i(x_{q2})$ , or equivalently

$$\begin{aligned} & c_{i0} + c_{i1}f_1(x_{q1}) + \dots + c_{ip}f_p(x_{q1}) \\ & > c_{i0} + c_{i1}f_1(x_{q2}) + \dots + c_{ip}f_p(x_{q2}). \end{aligned} \quad (4)$$

This inequality provides useful information for determining the parameters  $c_{ij}, j = 0, \dots, p$ . However, the inequality does not have any information to determine  $c_{i0}$  since any value of  $c_{i0}$  satisfies it. To remove these excessive degrees of freedom,  $c_{i0}$  is subtracted from the both sides of inequality (4), and the both sides are divided by  $c_{i1}$ . Here remember that  $c_{i1}$  is negative. Then the above inequality is rewritten as

$$\begin{aligned} & f_1(x_{q1}) + \frac{c_{i2}}{c_{i1}}f_2(x_{q1}) + \dots + \frac{c_{ip}}{c_{i1}}f_p(x_{q1}) \\ & < f_1(x_{q2}) + \frac{c_{i2}}{c_{i1}}f_2(x_{q2}) + \dots + \frac{c_{ip}}{c_{i1}}f_p(x_{q2}). \end{aligned} \quad (5)$$

By defining new preference parameters  $\alpha_{ij}$  as

$$\alpha_{ij} = \frac{c_{ij}}{c_{i1}}, \quad j = 2, \dots, p, \quad (6)$$

the following inequality (7) gives a condition that the parameters  $\alpha_{ij}, j = 2, \dots, p$ , must satisfy:

$$\begin{aligned} & (f_2(x_{q1}) - f_2(x_{q2}))\alpha_{i2} + \dots + (f_p(x_{q1}) - f_p(x_{q2}))\alpha_{ip} \\ & < f_1(x_{q2}) - f_1(x_{q1}). \end{aligned} \quad (7)$$

This linear inequality in  $\alpha_{ij}, j = 2, \dots, p$ , implies that in the  $(p-1)$ -dimensional parameter space which has  $\alpha_{i2}, \dots, \alpha_{ip}$ -axes, the true parameter vector  $[\alpha_{i2} \dots \alpha_{ip}]^T$  lies in one of the two halves of the space separated by the boundary hyper-plane

defined by

$$\begin{aligned} & (f_2(x_{q1}) - f_2(x_{q2}))\alpha_{i2} + \dots + (f_p(x_{q1}) - f_p(x_{q2}))\alpha_{ip} \\ & = f_1(x_{q2}) - f_1(x_{q1}). \end{aligned} \quad (8)$$

Note that every parameter  $\alpha_{ij}, j = 2, \dots, p$ , is positive because  $c_{i1}$  and  $c_{ij}$  are negative. Combining these positiveness conditions with the above inequality and, if necessary, appropriately large upper bounds  $\bar{\alpha}_{ij}$  of  $\alpha_{ij}$ , the existence region of the true parameter vector can be obtained which is a  $(p-1)$ -dimensional hyper-polyhedron. Figure 1 illustrates this for a two-dimensional ( $p = 3$ ) case where the existence region is a polygon. The true parameter vector lies somewhere in this polyhedron. There is no additional information to find where exactly, and therefore, the center of this hyper-polyhedron is adopted as the estimate of the true parameter vector:

$$\begin{bmatrix} \hat{\alpha}_{i2} \\ \vdots \\ \hat{\alpha}_{ip} \end{bmatrix} = \frac{1}{N} \sum_{k=1}^N \mathbf{u}_k, \quad (9)$$

where  $\mathbf{u}_k$  is  $k$ -th vertex of the hyper-polyhedron and  $N$  is the number of vertexes. The vertexes can be obtained by a method similar to Simplex method for Linear Programming problems because the hyper-polyhedron is determined by a set of linear inequalities with respect to  $\alpha_{ij}, j = 2, \dots, p$ .

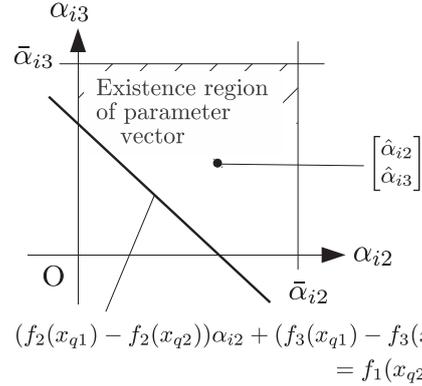


Fig. 1. Existence region of preference parameter vector for the case of  $p = 3$ .

In the second step, parameters  $c_{ij}, j = 0, \dots, p$ , are estimated. According to the definition of  $\alpha_{ij}$ ,  $v_i$  can be written as

$$v_i = c_{i0} + c_{i1}(f_1 + \alpha_{i2}f_2 + \dots + \alpha_{ip}f_p). \quad (10)$$

Since we already have estimates of  $\alpha_{ij}, j = 2, \dots, p$ , and values of  $f_j, j = 1, \dots, p$  are all available, we can estimate parameters  $c_{i0}$  and  $c_{i1}$  only if the values of evaluator's  $v_i$  are obtained. Here, the linguistic preference value included in the answer from the evaluator is used. It is converted to a corresponding numerical value and is used, together with  $v_i$  values obtained in all the previous interaction rounds, to estimate  $c_{i0}$  and  $c_{i1}$  by the least square method. Once the estimate  $\hat{c}_{i1}$  of  $c_{i1}$  is obtained, estimates of other  $c_{ij}$  can be derived as

$$\hat{c}_{ij} = \hat{c}_{i1}\hat{\alpha}_{ij}, \quad j = 2, \dots, p. \quad (11)$$

Note that in the very first round of interaction, only a single value of  $v_i$  is available, which is not sufficient to estimate two parameters  $c_{i0}$  and  $c_{i1}$ . Therefore their estimation starts at the second round.

At a round  $r$  of interaction, the current estimates of  $c_{i0}, \dots, c_{ip}, i = 1, \dots, m$  give the current estimate of the individual preference functions,  $\hat{v}_i^r(x), i = 1, \dots, m$ . These are combined into the current estimate of collective preference,  $\hat{V}^r(x)$ . Then, the tentative solution  $\hat{x}^r$  is derived by maximizing  $\hat{V}^r(x)$ . This tentative solution is used as one of the two alternatives presented to the evaluators in the question at the next round  $r+1$ ,  $x_{q1}^{r+1}$ . The other alternative  $x_{q2}^{r+1}$  is chosen so that the boundary hyperplane defined by  $x_{q1}^{r+1}$  and  $x_{q2}^{r+1}$  as in eq. (8) intersects with the current existence region of the preference parameter vector of an evaluator  $i$ ,  $R_i^r$ , as shown in Fig.2. Then, the existence region will be cut off and reduced at the next round. If there is more than one alternative that satisfies this requirement, then one alternative is selected so that it has one of the following features: (1) the boundary hyper-plane formed by  $x_{q1}^{r+1}$  and  $x_{q2}^{r+1}$  divides the existence region  $R_i^r$  into two parts of approximately equal volumes, (2) the selected alternative is closest to  $x_{q1}^{r+1}$ , and (3) the selected one has the highest value of  $\hat{V}^r$  except for  $x_{q1}^{r+1} = \hat{x}^r$ . The first feature (1) is effective for efficiently reducing the size of existence region. The latter two features mean that  $x_{q1}^{r+1}$  and  $x_{q2}^{r+1}$  should be close to each other in some sense. This is because the individual preferences are approximately expressed by tangential hyper-planes in the  $f$ -space which are valid in a small region and we want to estimate the preferences around the best solution. Therefore the distance between  $x_{q1}^{r+1}$  and  $x_{q2}^{r+1}$  is measured in the  $f$ -space but not in the  $x$ -space. If no such  $x_{q2}^{r+1}$  exists for the evaluator  $i$ , then another evaluator will be tried.

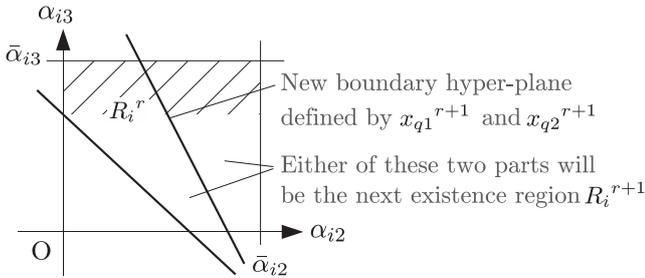


Fig. 2. New hyper-plane cutting off the current existence region of preference parameter vector.

### C. Termination of interactions and determination of the best solution

At each round of interaction, the two steps are executed consecutively. By repeating the rounds with different pairs  $x_{q1}$  and  $x_{q2}$  for questions, the existence region of the true parameter vector  $[\alpha_{i2} \dots \alpha_{ip}]^T$  is gradually cut off by the new boundary hyper-plane and reduced in size at each round, and a more accurate estimate is obtained. The interaction ends when no more question exists that can cut off the existence region.

Note that there are a finite number of alternatives  $x_1, \dots, x_n$  and that the interaction terminates in a finite number of rounds. The tentative solution derived at the final interaction round is adopted as the best solution  $\hat{x}$ .

### D. Procedure

The procedure of the method described so far is summarized as follows.

- 1) Set the interaction round index  $r = 0$ . The initial existence region of the preference parameter vector,  $R_i^0$ , of every evaluator  $i$  is an open sector defined by  $\alpha_{ij} > 0, j = 2, \dots, p$ . Or, if the upper bounds  $\bar{\alpha}_{ij}$  of  $\alpha_{ij}$  are given for  $j = 2, \dots, p$ ,  $R_i^0$  is a hyper-polyhedron.
- 2) Select  $x_{q1}^1$  either randomly or based on some *a priori* knowledge. Then, choose  $x_{q2}^1$  so that the hyper-plane defined by eq. (8) goes through the current existence region of the preference parameter vector of evaluator 1,  $R_1^0$ . If more than one alternative exists that satisfies this requirement, choose the one closest to  $x_{q1}^1$ . Set  $r = 1$ .
- 3) Present the pair of alternatives  $x_{q1}^1$  and  $x_{q2}^1$  to every evaluator. Find which each evaluator prefers and obtain the linguistic evaluation of the better alternative of the two. For each evaluator, if the hyper-plane cuts off his/her current existence region  $R_i^0$ , then obtain the new region  $R_i^1$ , otherwise let  $R_i^1 = R_i^0$ .
- 4) Calculate the parameter estimates  $\hat{\alpha}_{ij}^1, j = 2, \dots, p$ , for every evaluator  $i$ .
- 5) Select  $x_{q1}^2$  either randomly or based on some *a priori* knowledge. Then, choose  $x_{q2}^2$  so that the hyper-plane defined by eq. (8) intersects with the current existence region of evaluator 2,  $R_2^1$ . If more than one alternative exists that satisfies this requirement, choose the one closest to  $x_{q1}^2$ . Set  $r = 2$ .
- 6) Present the alternatives  $x_{q1}^r$  and  $x_{q2}^r$  to every evaluator. Find which each evaluator prefers and obtain the linguistic evaluation of the better alternative. For each evaluator, if the hyper-plane severs his/her current existence region  $R_i^{r-1}$ , then obtain the new region  $R_i^r$ , otherwise let  $R_i^r = R_i^{r-1}$ .
- 7) Calculate the parameter estimates  $\hat{\alpha}_{ij}^r, j = 2, \dots, p$ , for every evaluator  $i$ . Then, find the estimates  $\hat{c}_{ij}^r, j = 0, \dots, p$ , using the  $v_i$  values obtained up to the current round. Now the current estimate of collective preference function  $V^r(x)$  is available.
- 8) Find the tentative solution  $\hat{x}^r$  by maximizing  $V^r(x)$ .
- 9) Set  $x_{q1}^{r+1} = \hat{x}^r$ . Determine  $x_{q2}^{r+1}$  so that the boundary hyper-plane defined by  $x_{q1}^{r+1}$  and  $x_{q2}^{r+1}$  goes through the current existence region  $R_{i_t}^r$  of evaluator  $i_t$ , where  $i_t = \{(r-1) \bmod p\} + 1$ . If more than one of such  $x_{q2}^{r+1}$  exists, choose the one that has one of the features described in Subsection III-B. If such  $x_{q2}^{r+1}$  does not exist, try the current existence regions of evaluator  $i_t + 1, i_t + 2$  and so on. If such  $x_{q2}^{r+1}$  does not exist for any evaluator, then the procedure terminates

and the current tentative solution  $\hat{x}^r$  is adopted as the best solution  $\hat{x}$ .

10) Increment  $r$  by 1 and go back to step 6).

#### E. Remarks

The individual preference estimation problem treated here has some incompleteness. Firstly, the function form of the preference function  $v_i(x)$  to be estimated is not well known. Therefore the unknown preference function is approximated by a tangential hyper-plane in the  $f$ -space. And secondly, the set of data to be used for estimation is poor in both quality and quantity since the evaluators cannot give precise and accurate numerical preference values and it is not realistic to ask questions repeatedly many times to the evaluators (clients). The tangential hyper-plane expression is also advantageous to deal with the poor quantity of data since the number of parameters to be estimated is reduced. The tangential hyper-plane is, however, a local expression. Its parameters should be estimated based on the ‘local’ data in the neighbourhood of the tangent point. Still, since we do not have many data, we have to use all the data obtained so far which may include ‘non-local’ data as well. This is justified by the safe assumption that the preference functions in our minds are not multi-modal and do not violently fluctuate when  $f_j$  values change only slightly. Therefore the tangential hyper-plane expression should be valid over a relatively wide region in the  $f$ -space.

Although the evaluators cannot give precise and accurate numerical preference values, it is expected that they can compare and distinguish two alternatives. Therefore, these ‘pairwise comparisons’ give much accurate information on the evaluator’s preferences. In the method, the information from pairwise comparisons is dominantly used in the preference parameter estimation. And, the imprecise linguistic preference information is employed for estimation of just two parameters,  $c_{i0}$  and  $c_{i1}$ . In this way, the possible undesirable effects caused by imprecision in the evaluators’ answers are alleviated.

#### IV. EXAMPLE

The proposed method has been applied to an example problem which has twenty alternatives, three attributes and three evaluators. The attribute criterion values are listed in Table I. In order to control the problem, the human evaluators are replaced with their models. Each evaluator  $i$  is modeled by a preference function  $v_i$  which is assumed to be exactly represented by eq. (1). Three different cases are considered. In Case 1, all the three evaluators have similar preference parameters  $c_{ij}, i = 1, 2, 3; j = 0, \dots, 3$ . In Case 2, evaluator 1 and 3 have similar preferences but evaluator 2 has a different one. In Case 3, the preferences of the three evaluators are all different. The preference parameters are normalized so that the  $v_i$  values range from 0.5 to 5.4. When linguistic values are necessary, the  $v_i$  values are rounded to the nearest integers, which means that five linguistic values are considered here. The collective preference is defined as  $V(s) = v_1(s) + v_2(s) + v_3(s)$ . The preference parameters and their corresponding true best solution  $x^*$  are listed in Table II.

TABLE I  
ATTRIBUTE CRITERION VALUES OF TWENTY ALTERNATIVES.

Solution	$f_1$	$f_2$	$f_3$
$x_1$	3	400	14.0
$x_2$	4	300	11.4
$x_3$	5	375	7.4
$x_4$	6	275	7.8
$x_5$	6	325	6.7
$x_6$	7	200	10.0
$x_7$	8	250	6.3
$x_8$	8	350	4.5
$x_9$	9	200	7.4
$x_{10}$	9	400	3.6
$x_{11}$	10	275	4.3
$x_{12}$	11	150	8.7
$x_{13}$	11	225	5.1
$x_{14}$	11	325	3.1
$x_{15}$	13	175	6.3
$x_{16}$	13	275	3.3
$x_{17}$	13	375	2.2
$x_{18}$	14	325	2.4
$x_{19}$	15	100	14.0
$x_{20}$	15	225	4.3

TABLE II  
EXAMPLE SPECIFICATIONS.

Case	$c_{i0}$	$c_{i1}$	$c_{i2}$	$c_{i3}$	$x^*$
1	17.44	-0.481	-0.0265	-0.337	$x_6$
	17.73	-0.489	-0.0259	-0.352	
	16.96	-0.458	-0.0261	-0.312	
2	17.20	-0.443	-0.0256	-0.365	$x_9$
	10.91	-0.197	-0.0056	-0.492	
	17.56	-0.464	-0.0244	-0.422	
3	18.19	-0.639	-0.0134	-0.378	$x_7$
	10.91	-0.197	-0.0056	-0.492	
	17.56	-0.464	-0.0244	-0.422	

In the table, each case has three rows, and each row lists the preference parameters of each evaluator. The alternatives to be used as questions are selected so that they have the feature (1) described in Subsection III-B.

The obtained results are shown in Table III. In all of the three cases with different evaluator preference distributions, the true best solution  $x^*$  is successfully obtained as the solution  $\hat{x}$ . The preference parameter estimates are in the range from 65% to 137% of their corresponding true values. Note that, since there are only a finite number of alternatives, the interaction process terminates before the existence regions of the parameter vector converges to single points and that the parameter estimates cannot converge to their true values. Considering also that the linguistic values are used for their estimation, these values can be regarded as good estimates. The final solutions are derived after five to eleven rounds of interaction, which is small considering there are a total of  $\binom{20}{2} = 190$  possible pairs of alternatives to be presented as questions. This illustrates that the proposed method can find the best solution efficiently.

#### V. CONCLUSIONS

A solution method has been proposed that addresses a class of multiple-attribute decision making problems where the alternatives are evaluated by multiple evaluators different

TABLE III  
EXAMPLE RESULTS.

Case	$\hat{c}_{i0}$	$\hat{c}_{i1}$	$\hat{c}_{i2}$	$\hat{c}_{i3}$	$\hat{x}$	No. of rounds
1	18.33	-0.636	-0.0237	-0.368	$x_6$	5
	16.61	-0.567	-0.0211	-0.328		
	12.12	-0.348	-0.0130	-0.368		
2	16.03	-0.361	-0.0220	-0.437	$x_9$	9
	9.14	-0.129	-0.0039	-0.439		
	18.78	-0.451	-0.0280	-0.444		
3	14.94	-0.580	-0.0150	-0.247	$x_7$	11
	10.73	-0.177	-0.0061	-0.471		
	19.53	-0.400	-0.0340	-0.550		

from the decision maker. Two major issues in solving these problems are resolved by use of a set of predetermined linguistic values and their corresponding numerical values for the preference representation and the two-step estimation of evaluators' preferences based on easy-to-answer questions. The example illustrates that the proposed method can successfully find the best solution and good preference parameter estimates with a small number of iteration rounds.

It has been assumed that the evaluators give correct and consistent answers in the interactions. However this is not always the case in reality. Some people may give incorrect or inconsistent answers, and even worse some may not give answers. Investigation into the effects caused by these incomplete answers and, if necessary, an improvement of the method will be future work. Also, the proposed method assumes that each evaluator can be identified throughout the rounds of interactions so that the method can estimate the individual preferences separately. In practice, the clients of public services come in and go out as they wish, and it is not easy to keep track of individuals over a long time. A method that can deal with more number of evaluators with

no identifications will be necessary. Extension of the current method in this direction is now underway.

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