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MULTIPLE-ATTRIBUTE DECISION MAKING WITH INTERACTIVE ESTIMATION OF USER PREFERENCE

Junichi Murata and Kentaro Kitahara
Department of Electrical Engineering
Kyushu University
Fukuoka, Japan
email: murata@ees.kyushu-u.ac.jp

ABSTRACT

The aim of multi-objective optimization or multiple-attribute decision making is to find the most appropriate solution among Pareto solutions. Usually, decision makers are in charge of evaluating and rating objective functions or attributes. However, in a good number of practical decision making problems especially in the area of public services e.g. determining the service contents of public transport, the decision makers determine the service contents while the users evaluates the quality of the service. In these problems where decision makers and evaluators are different, an effective method is necessary that can estimate the preferences of the evaluators (users). In the paper, an interactive method is proposed that estimates or learns the user preferences and finds the solution based on the estimated results.

KEY WORDS

multiple-attribute decision making, Multi-objective optimization, Interactive methods, Learning, Public services.

1 Introduction

The aim of multiple-attribute decision making (MADM), and multi-objective optimization (MOOP) and multiple criteria decision making (MCDM) alike, is to find the most appropriate solution among a number of alternative solutions each of which has multiple attributes or is evaluated by multiple criteria (objective functions). Usually there is no complete optimal solution that is the best with respective to all the attributes, but there are a number of Pareto solutions optimal in the sense that there are no other solutions better than that in every attribute. Mathematically all the Pareto solutions are equal, and the decision maker must select a particular Pareto solution based on his/her preference. Therefore, the decision maker intervenes the optimization process. There have been proposed a number of methods to find the appropriate solution with the intervention by the decision maker before, during and after the optimization process [1]. Several weighted scalarizing methods of objective functions incorporate the decision maker's preference before the optimization [2],[3]. In the interactive methods, the decision maker intervenes during the optimization process, i.e. a question is posed based on the current solutions to the decision maker to derive information on his/her preference, and the answer is used to improve the solutions [4]. Recently extensive developments have been done on the last group of methods based on evolutionary computations, which provide the decision maker with a good number of Pareto solutions for his/her evaluation [5].

In all the above mentioned methods, it is assumed that the decision maker evaluates and rates the alternative solutions. In practice, there are a number of decision making problems where the evaluation and rating is done by a different person or people. A typical example is a public transport service [6]. The content of bus services is determined by the service operator who is the decision maker. There are several attributes of the services such as vehicle scheduling and crew management that are the operator's concerns. But the quality of the services such as travelling times, service frequency and fares are evaluated by the passengers who cannot take part in the decision making and whose preferences are not well known by the decision maker. For this kind of problems, a method is necessary which can estimate the user preferences and use them to find the best solutions. In this paper such a method is proposed. Note that the user preferences are not well known in advance and that it is impractical or impossible to present many Pareto solutions to the users and ask for their evaluations. Therefore, the interactive method is most promising. In the existing interactive methods, the interactions or questions-and-answers are performed between the decision maker and the aiding system such as optimization software tool. So, the answers can be derived immediately after the questions are presented to the decision maker who is willing to give accurate and coherent answers. On the other hand, in our target problems, the interaction must be done between the decision maker (with the aid of the software tool) and the users. There can be many users. They can be in remote places. They can be unhelpful. So, the interaction must be simple and efficient so that the users can take part in the interaction without much difficulty and can give accurate and coherent answers. It is also necessary that the method can deal with multiple users. Asking many users many questions and collecting their answers will require a lot of time and cost. So, the interactions should be simple and efficient from not only the users' viewpoint but also from the decision maker's.

Here in this paper, an interactive method is proposed for estimation or learning of the preference of a single user in a simple and efficient way that does not impose much burden on the user. The method, as the first step toward the aim of the research, does not treat multiple users, but its simplicity is advantageous when extending to the problems involving multiple users.

The rest of paper is organized as follows. In the next section, the problem treated here is stated. The ideas underlying the proposed method are described in Section 3 which is followed by the specific procedure. The validity of the procedure is illustrated by examples described in Section 4. Then concluding remarks are given in Section 5.

2 Problem Statement

There are n alternative solutions $x_1, \cdots, x_n$ each of which has k attributes. Goodness of each attribute is measured by its evaluation criterion. Here we denote an evaluation criterion for the j-th attribute by $f_j(\cdot)$ which, we assume, takes positive values and is to be minimized. For simplicity we treat an attribute j of a solution x_i and its criterion value $f_j(x_i)$ interchangeably. We assume that there is a single evaluator (user). It is also assumed that the evaluator or the user well understands his/her own preference v so that (s)he can give correct and consistent answers to the questions raised by the decision maker during the interaction process. The preference v is a function of $f_1(x), \cdots, f_k(x)$ and is assumed to be maximized.

3 Multiple-Attribute Decision Making with Interactive Estimation of User Preference

The easiest way for a user to give information on his/her preference should be choosing preferable one from two alternative solutions [7],[8]. Selecting one among three or more alternatives would be difficult, and answering his/her preference focusing on a single attribute ignoring the others would be also hard. From the decision maker's point of view, posing different questions to each of many users would require a lot of time and cost. Asking the users the same question at the same time will be easier. One possible way for this in public services domain would be providing the users with two different services one after another and asking for their comparisons. The decision maker (the service operator) does not need to do anything special, e.g. distributing questionnaire sheets, in order to ask the question. In this case, the users answer their preferences based on their actual experiences of the services. So, it is desirable that the services to be experienced by the users in the interaction process are good ones. Here in this paper, an interactive method is proposed which employs the above easy-to-answer questions. We treat problems with a single user only. However, we keep it in mind that the series of questions to be presented to the user should be good for the user so that the experience-based interaction becomes feasible and that the method can be extended to multiple users cases.

In the proposed method, the user preference is approximately represented by tangential hyper-planes of indifferent (equi-preference) hyper-surfaces. The tangential planes are updated whenever the decision maker obtains the user's answer to a question presented by the decision maker. Then the tentative best (most appropriate) solution is found as an optimum of the approximately estimated preference, which is in turn used to compose the next question to be posed to the user. In this way, estimation of the user preference and finding the tentative best solution are executed alternatingly toward the best solution through the interactions or questions-and-answers between the decision maker and the user. In the following, the approximate representation of user preference is described first, and then its estimation or learning method will be explained.

3.1 Representation of User Preference

Figure 1 shows a typical contour map of the user preference v on f_1-f_2 plane for the case of $k = 2$. Since the criteria $f_j, j = 1, \cdots, k$, are to be minimized, the preference v is higher for the contours closer to the origin. The contours do not cross with each other and thus, in a small region, can be regarded as parallel. Then, in a small region, the user preference v can be approximately represented by a set of parallel tangential lines of the contours (see Fig.1). For general k, the user preference v can be approximately represented by a set of parallel tangential hyper-planes of the contours or indifferent hyper-surfaces. The tangential hyper-plane is expressed in the following form:

$$c_0 + c_1 f_1(x) + \cdots + c_k f_k(x) = v. \tag{1}$$

Since the preference v increases monotonically as any of f_j, $j = 1, \cdots, k$, decreases, the coefficients $c_1, \cdots, c_k$ are all negative. Considering that c_0 is a constant and that $c_1 < 0$, maximizing the user preference v is equivalent to maximizing $c_1 f_1(x) + \cdots + c_k f_k(x)$, and is also equivalent to minimizing

$$\begin{aligned} F(x) &= f_1(x) + \frac{c_2}{c_1} f_2(x) + \cdots + \frac{c_k}{c_1} f_k(x) \\ &= f_1(x) + \alpha_2 f_2(x) + \cdots + \alpha_k f_k(x). \end{aligned} \tag{2}$$

Since all of the coefficients c_j are negative, the parameters α_j are positive. Once we know the values of parameters $\alpha_2, \cdots, \alpha_k$, we have an approximate representation of the user preference which is valid in a small region of the criterion (f_j) space. Note that minimization of $F(x)$ is equivalent to the weighting method for solving multi-objective optimization problems and gives a Pareto solution since the parameters or weights α_j are all positive [1].

The user preference is unknown and therefore we must estimate the true values of the parameters, $\alpha_2^*, \cdots, \alpha_k^*$, based on the information derived by the interactions between the decision maker and the user. The

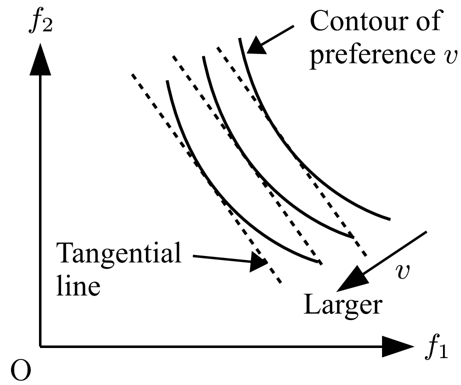


Figure 1. Contours and tangential lines of typical user preference ($k = 2$).

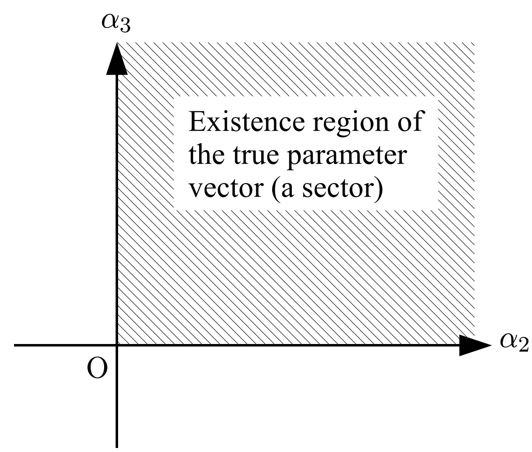


Figure 2. Region where the true parameters exist ($k = 3$).

approximate representation of preference is valid only in a small region. Therefore, we must know the user's preference information near the best solution. Of course we do not know where the best solution is at the beginning of the interaction process. So, it should be reasonable to estimate the parameters which are valid in a certain small region based on the user's current answer, find the tentative best solution using the estimates, and compose the next question to be given to the user so that we can know about the user preference in a better small region. This alternation of estimation of the preference and finding the tentative best solution has another advantage. In this method, the user does not have to give exhaustive information about his/her preference at once, which will be a large burden on the user, but is repeatedly asked small questions. Therefore, the burden on the user is not heavy, and it will be easy to obtain coherent answers from the user. Moreover, the questions are updated based on the latest estimate of the preference just like a hill-climbing method. The process cannot be proved to converge to the true/best estimate/solution, but is efficient since it searches promising area only. The estimation procedure is described in the next subsection.

3.2 Estimation of User Preference

Let us consider the $(k-1)$-dimentional parameter (α_j) space. All the parameters α_j are positive. Then we can say, without any prior information on the user preference, that the true parameter vector $[\alpha_2^* \cdots \alpha_k^*]^{\mathrm{T}}$ which represents the user's actual preference lies somewhere in the sector defined by $\alpha_2 > 0, \cdots, \alpha_k > 0$ as shown in Fig.2 for the case of $k = 3$. We want to narrow down this sector-shaped existence region of the true parameter vector based on the information derived through the interactions.

In a single round of interactions, the decision maker presents two solutions, say, x_{q_1} and x_{q_2}, to the user. The user answers which of the two is better according to his/her own preference. If (s)he answers that x_{q_1} is better than x_{q_2}, it means that $F(x_{q_1}) < F(x_{q_2})$, or

$$\begin{aligned} & f_1(x_{q_1}) + \alpha_2^* f_2(x_{q_1}) + \cdots + \alpha_k^* f_k(x_{q_1}) \\ & \quad < \quad f_1(x_{q_2}) + \alpha_2^* f_2(x_{q_2}) + \cdots + \alpha_k^* f_k(x_{q_2}), \end{aligned} \tag{3}$$

which can be rewritten as a condition on the true parameters α_j^*:

$$\begin{aligned} & (f_2(x_{q_2}) - f_2(x_{q_1}))\alpha_2^* + \cdots + (f_k(x_{q_2}) - f_k(x_{q_1}))\alpha_k^* \\ & \qquad > f_1(x_{q_1}) - f_1(x_{q_2}). \end{aligned} \tag{4}$$

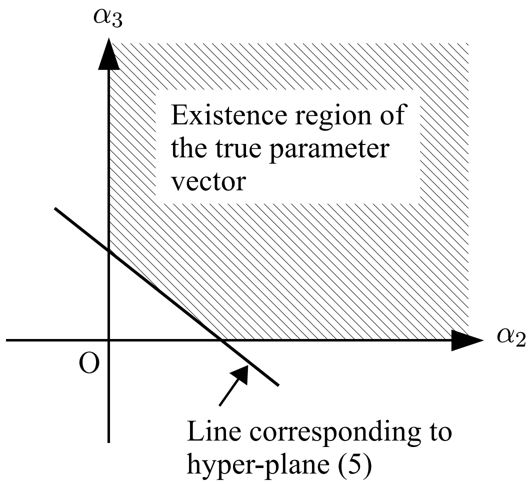
The above inequality implies that the true parameter vector $[\alpha_2^* \cdots \alpha_k^*]^{\mathrm{T}}$ lies in one of the two halves of the parameter space divided by the hyper-plane defined by

$$\begin{aligned} & (f_2(x_{q_2}) - f_2(x_{q_1}))\alpha_2 + \cdots + (f_k(x_{q_2}) - f_k(x_{q_1}))\alpha_k \\ & \qquad = f_1(x_{q_1}) - f_1(x_{q_2}). \end{aligned} \tag{5}$$

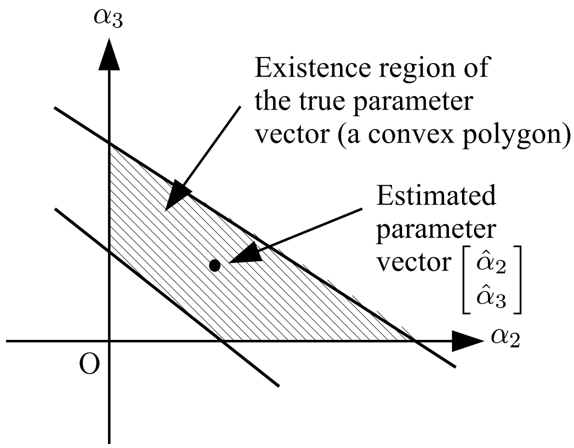
The hyper-plane cuts off the sector, and we now have a smaller existence region of the true parameter vector. By repeating the rounds of interactions and obtaining user's answers to different questions, the sector is repeatedly severed by different hyper-planes. Then the existence region of the true parameter vector becomes a convex hyper-polyhedron. It should be natural to pick up a center of the existence region as the estimate of the true parameter vector. Here, the estimate vector $[\hat{\alpha}_2 \cdots \hat{\alpha}_k]^{\mathrm{T}}$ of the true parameter vector is calculated as the center of the vertices of the hyper-polyhedron. The process is illustrated in Fig.3.

If the user answers that the solutions x_{q_1} and x_{q_2} are equal according to his/her preference, the true parameter vector exists on the hyper-plane (5) clipped by the sector or the existence region determined at the previous round of interactions. Then the same procedure will continue on this existence region.

The pairs of solutions x_{q_1} and x_{q_2} to be presented to the user must be selected so that they form the boundary hyper-plane (5) running through the hyper-polyhedron. On the other hand, we want to estimate the user preference in



(a) After the first cutting off.



(b) After the second cutting off.

Figure 3. Shrinkage of existence region of the true parameter vector.

the region near the (unknown) best solution. So, it is desirable that either x_{q_1} or x_{q_2} or both are selected to be closer to the best solution. When there are no pair of solutions whose corresponding boundary hyper-plane intersects the current existence region, we cannot shrink the region any more, and the procedure terminates.

3.3 Proposed Procedure

Based on the idea described in the previous subsection, the specific procedure of estimation of the user preference and finding the best solution will be described below. To begin with, let us define some symbols as follows:

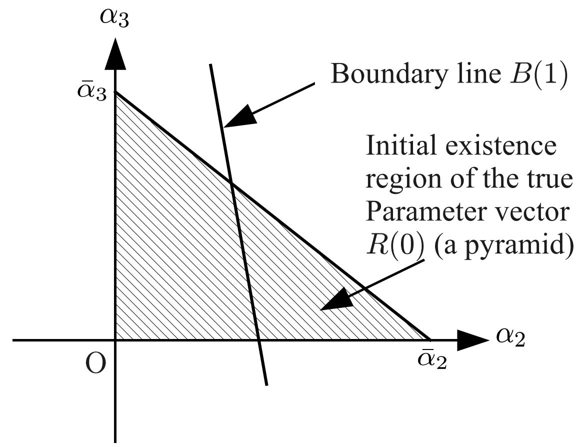


Figure 4. Initial existence region of the true parameters and its cutting ($k = 3$).

l	:	interaction round (iteration),
$x_{q_1}(l), x_{q_2}(l)$	:	two solutions presented to the user at round l,
$B(l)$	:	boundary hyper-plane defined by the solutions $x_{q_1}(l)$ and $x_{q_2}(l)$,
$R(l)$	:	existence region of the true parameters derived at round l,
$\boldsymbol{u}_p(l)$	:	$(k-1)$-dimensional vector expressing p-th vertex of the existence region $R(l)$,
$n(l)$	:	the number of vertices that the existence region $R(l)$ has,
$E(\boldsymbol{u}_p(l), \boldsymbol{u}_q(l))$	:	an edge of the existence region $R(l)$ with the endpoints $\boldsymbol{u}_p(l)$ and $\boldsymbol{u}_q(l)$.

First, based on some prior knowledge, the decision maker gives the upper bounds $\bar{\alpha}_j$ on the parameters $\alpha_j, j = 2, \cdots, k$. If there is no prior information, the upper bounds are selected to be sufficiently large values. Now, the initial existence region $R(0)$ of the true parameters α_j^* is not an infinite sector but a finite pyramid (a hyper-polyhedron) as shown in Fig.4.

Then the first round of interaction starts. The decision maker presents a pair of solutions $x_{q_1}(1)$ and $x_{q_2}(1)$ to the user as the question. One of the solutions, say $x_{q_1}(1)$, is selected based on some prior information, if any, or, otherwise, randomly. The other solution $x_{q_2}(1)$ is selected so that the boundary hyper-plane $B(1)$ composed by $x_{q_1}(1)$ and $x_{q_2}(1)$ intersects the initial existence region $R(0)$. In general, the existence region is a convex hyper-polyhedron, and therefore the existence of intersection is judged by checking if there is at least one vertex of the hyper-polyhedron $R(l-1)$ lying on one side of the boundary hyper-plane $B(l)$ and at least one vertex on the other side. If there are two or more solutions that satisfy the intersecting conditions, then the solution nearest to $x_{q_1}(1)$ in the criterion space is adopted as $x_{q_2}(1)$. This is because we are estimating the user preference that is valid in a small region. The following scaled distance is used to measure the distances in the criterion space:

$$d(x_{q_1}, x) = \sqrt{\sum_{j=1}^{k} w_j (f_j(x_{q_1}) - f_j(x))^2}, \qquad (6)$$

where $w_j, j = 1, \cdots, k$, are weights that adjust the differences in the value ranges of $f_1, \cdots, f_k$.

The above steps of the estimation procedure are specific to the first round ($l = 1$). The remaining steps are, however, applied to every round including the first round, and therefore they are described assuming that the current round is l.

The two solutions are presented to the user, and the user answers which is better. Unless the user answers that both are equally preferable, the existence region determined in the previous round $R(l-1)$ is cut off by the boundary hyper-plane $B(l)$ to give the new smaller existence region $R(l)$. If the both solutions are judged to be equal by the user, then the intersection of the boundary hyper-plane $B(l)$ and the previous existence region $R(l-1)$ becomes the new existence region $R(l)$. The procedure for cutting off the existence region described in Appendix can be used for the cases of k up to four. Or alternatively, all the vertices of $R(l)$ can be calculated by noting that the inequalities (4) obtained up to this round altogether determine $R(l)$. Because (4) is a linear inequality, determining vertices of $R(l)$ is identical to calculation of the extreme points of feasible regions in linear programming problems.

The center of the vertices of the new existence region $R(l)$ is adopted as the estimate of the true parameter vector:

$$\begin{bmatrix} \hat{\alpha}_2(l) \\ \vdots \\ \hat{\alpha}_k(l) \end{bmatrix} = \frac{1}{n(l)} \sum_{p=1}^{n(l)} \boldsymbol{u}_p(l). \qquad (7)$$

Now, we can find the tentative best solution by minimizing

$$\hat{F}^l(x) = f_1(x) + \hat{\alpha}_2(l) f_2(x) + \cdots + \hat{\alpha}_k(l) f_k(x). \qquad (8)$$

The derived tentative best solution $\hat{x}(l)$ is used as one of the solutions to be presented to the user in the next round, $x_{q_1}(l+1)$. The other one $x_{q_2}(l+1)$ is selected so that the boundary hyper-plane that these two solutions define, $B(l+1)$, intersects the current existence region $R(l)$. Note that we treat multiple-attribute decision making problems here and that therefore we have a finite number of alternative solutions. So, at a certain round, there will be no pair of solutions left that gives such a boundary hyper-plane, implying that we cannot reduce the existence region any more. The procedure terminates here with the current tentative best solution $\hat{x}(l)$ as the final solution $\hat{x}$. When we have two or more solutions that satisfy the intersecting condition, then we pick up, as $x_{q_2}(l+1)$, either the one simply nearest to $x_{q_1}(l+1)$ or the nearest to $x_{q_1}(l+1)$ among those solutions with high preference or equivalently low $\hat{F}^l(x)$ values.

Then, l is incremented by one, the selected pair of solutions are presented to the user, and the same steps as the above are followed until there is no pair of solutions that give the hyper-plane intersecting the existence region.

Table 1. Criterion values of twenty solutions.

Solution	f_1	f_2	f_3
x_1	3	4.00	14.0
x_2	4	3.00	11.4
x_3	5	3.75	7.4
x_4	6	2.75	7.8
x_5	6	3.25	6.7
x_6	7	2.00	10.0
x_7	8	2.50	6.3
x_8	8	3.50	4.5
x_9	9	2.00	7.4
x_{10}	9	4.00	3.6
x_{11}	10	2.75	4.3
x_{12}	11	1.50	8.7
x_{13}	11	2.25	5.1
x_{14}	11	3.25	3.1
x_{15}	13	1.75	6.3
x_{16}	13	2.75	3.3
x_{17}	13	3.75	2.2
x_{18}	14	3.25	2.4
x_{19}	15	1.00	14.0
x_{20}	15	2.25	4.3

4 Examples

In order to evaluate the effectiveness of the proposed method, let us apply the method to the following problem. There are twenty alternative solutions, each of which has three attributes. An attribute is evaluated by a criterion. Table 1 shows the criterion values f_1, f_2 and f_3 for the twenty solutions. Five different sets of true parameters α_j^* are assumed. The basic performance can be evaluated from three aspects: derived best solution $\hat{x}$, parameter estimates $\hat{\alpha_2}$ and $\hat{\alpha_3}$, and the total number of rounds required L. These are listed in Table 2. The table shows that the method gives the exact best solution in all the cases irrespective of the true parameters α_j^* and the initial question $x_{q_1}(1)$. Also, the parameter estimates are close to the true values. Since the target problem has a finite number of solutions, we can obtain only a finite number of answers from the user and thus the parameter estimates cannot converge to the true values. The number of rounds required is around ten which can be compared to the total number, 190, of possible pairs of twenty solutions to be presented to the user as questions.

When the experience-based interaction is employed, the users experience a pair of different services provided by the service operator (the decision maker) and give their

Table 2. Example specifications and results.

Case	Specifications				Results			
	α_2^*	α_3^*	x^*	$x_{q_1}(1)$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{x}$	L
1	3.57	0.91	x_7	x_8	3.91	1.19	x_7	8
2	3.57	2.0	x_{11}	x_8	4.24	2.02	x_{11}	12
3	3.57	0.48	x_6	x_8	3.29	0.26	x_6	6
4	5.26	0.91	x_9	x_8	5.01	0.84	x_9	7
5	1.72	0.91	x_5	x_8	1.30	0.82	x_5	8
6	3.57	0.91	x_7	x_7	3.97	1.06	x_7	10
7	3.57	0.91	x_7	x_1	4.02	1.08	x_7	12

Table 3. Average percentile preferences of the solutions presented to the user.

Case	$\bar{v}_{\%}$
1	89.2
2	88.5
3	81.5
4	87.3
5	92.2
6	79.3
7	82.0

preferences to the decision maker. Here, the users would naturally not want to experience poor services even if they are for information gathering purpose and are to be improved later on. The next table lists percentile value $\bar{v}_{\%}$ of preference v averaged over the all the solutions presented to the user during the interactions. Here $\bar{v}_{\%}$ is calculated as

$$\bar{v}_{\%} = \frac{\bar{v} - v_{\min}}{v_{\max} - v_{\min}}, \tag{9}$$

where $\bar{v}$ is the average preference over the all the solutions presented to the user, and $v_{\min}$ and $v_{\max}$ are the minimum preference and the maximum preference among all the twenty alternative solutions, respectively. The table implies that the user is presented with solutions with high preferences (80 % or higher) on average. So, the user do not need to endure experiencing poor solutions.

5 Remarks and Conclusion

An interactive method has been proposed that estimates the user preference and finds the best solution for multiple-attribute decision making problems where the decision maker and the evaluator (user) are different. The method described here treats a single user, but has been developed so that it can be easily extended to the cases with many users. The proposed method interactively estimates the local property of the user preference, and therefore the user can easily answer the questions. There are a number of methods that estimates the local property (trade-off ratio) of the (decision maker's) preference [7],[8],[9],[10]. Among them, the methods in [7],[8],[10] require basically yes-or-no answers just like the method proposed here. However, these methods use more questions than the proposed method in each round of interaction, which guarantees the convergence. In our method, on the other hand, a focus is placed on the reducing the user's burden in the interactions, which makes it unable to prove the convergence (note that the derived solution is yet guaranteed to be a Pareto solution). However, in public services, there are many users, and they change in time. Then, the best solution, if any, can also change in time. So, seeking for the exactly best solution would not be so important as adapting to the changes. The next thing to be done is to find the solution based on multiple users' preferences with consideration of the robustness of the method against errors and inconsistency in the users' answers.

Appendix

The procedure for cutting off the existence region is described here. For the case of $k = 4$, the existence region is a three-dimensional polyhedron and the boundary hyperplane is reduced to an ordinary two-dimensional plane. The following procedure can be also applied to cases where k is three or lower by appropriately substituting the polyhedrons and the planes with their corresponding shapes of the suitable dimensionalities.

When the existence region $R(l-1)$ and the boundary plane $B(l)$ intersects, some edges of the polyhedron $R(l-1)$ intersect the boundary plane. And their intersecting points become vertices of the new existence region $R(l)$. The vertices of the old existence region $R(l-1)$ are deleted if they are on the side of the boundary plane $B(l)$ where the true parameter vector does not exist. Whether a vertex is on the 'wrong' side of the boundary hyper-plane or not can be determined by checking if its coordinates satisfy the inequality (4).

A polyhedron consists of a number of surfaces. Let us specify a surface by a sequence of its vertices which are arranged in the counterclockwise order of appearances. For example, a surface in Fig.5 is expressed by $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_5\}$. Two adjacent vertices in this sequence form an edge of the surface. Any point on an edge having two vertices $\boldsymbol{u}_p(l-1)$ and $\boldsymbol{u}_q(l-1)$ as its endpoints, $E(\boldsymbol{u}_p(l-1), \boldsymbol{u}_q(l-1))$, can be represented by the vector

$$\begin{bmatrix} \alpha_2 \\ \vdots \\ \alpha_k \end{bmatrix} = \beta \boldsymbol{u}_p(l-1) + (1-\beta)\boldsymbol{u}_q(l-1),\ 0 \le \beta \le 1. \tag{10}$$

If there exist a $\beta, 0 \le \beta \le 1$, that satisfies both the above equation (10) and equation (5) that defines the hyper-plane, then they intersect.

Consider, for example, a surface shown in Fig.5. Suppose that its two edges $E(\boldsymbol{u}_1, \boldsymbol{u}_2)$ and $E(\boldsymbol{u}_4, \boldsymbol{u}_5)$ intersect the boundary hyper-plane and denote the intersecting points as $\boldsymbol{u}_6$ and $\boldsymbol{u}_7$, respectively. Also suppose that among the endpoints of the intersected edges, $\boldsymbol{u}_1$ and $\boldsymbol{u}_5$

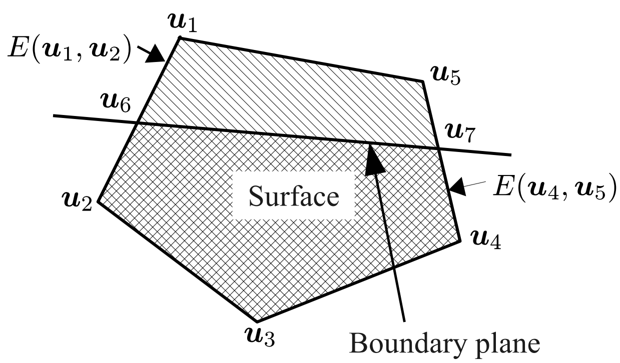


Figure 5. A surface of existence region and its cutting.

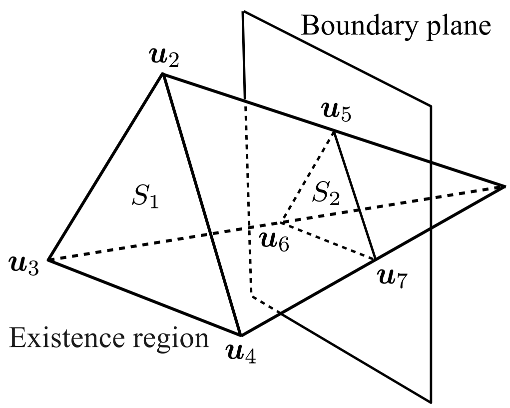


Figure 6. Cutting existence region by a boundary plane.

do not satisfy the inequality (4) expressing the user preference. The intersecting points are inserted to the ordered sequence of the vertices to give $\{\boldsymbol{u}_1, \boldsymbol{u}_6, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_7, \boldsymbol{u}_5\}$, and then the vertices on the wrong side of the boundary $\boldsymbol{u}_1$ and $\boldsymbol{u}_5$ are deleted, which results in the surface after cutting off as $\{\boldsymbol{u}_6, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_7\}$.

The new vertices introduced as the intersecting points themselves form a new surface. The order of those vertices is determined according to the order(s) of the vertices in the surface(s) consisting of only the endpoints of the intersected edges. In Fig.6, the new vertices $\boldsymbol{u}_5$, $\boldsymbol{u}_6$ and $\boldsymbol{u}_7$ form the new surface S_2, and their order is determined according to the order of vertices $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ and $\boldsymbol{u}_4$ contained in the surface S_1.

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